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Analytical Model for Momentum Transfer of Spacecraft Containing Liquid

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Nomenclature

C, C_1	=	centers of masses of the system and the main body
$\mathbf{E}$	=	3×3 identity matrix
$\mathbf{H}$, $\mathbf{h}_w$, $\mathbf{h}_p$	=	angular momenta of the system, the wheel, and the pendulum
$\hat{\mathbf{H}}$	=	angular momentum of the system as $\mu \to 0$
$\mathbf{I}_b$, $\mathbf{I}_w$	=	moments of inertia of the spacecraft main body and wheel
$\mathbf{J}$	=	moment of inertia including main body, wheel, and slosh mass
m_1, m_2	=	masses of spacecraft main body and pendulum bob
O	=	pivot point of the pendulum (center of tank location)
r	=	length of massless rod connecting the pendulum bob assumed constant
$\mathbf{r}$, $\mathbf{r}_o$, $\mathbf{r}_p$	=	position vectors from O to m_2, C_1 to O, and C_1 to m_2
r_o	=	distance from C_1 to O
t	=	time, s
$\mathbf{T}_o$	=	frictional torque about O
$\mathbf{u}_r$, $\mathbf{u}_p$	=	$\mathbf{r}/r$ and $\mathbf{r}_p/r$
$x_i y_i z_i$	=	body-fixed frame having its origin at C_i
$\boldsymbol{\alpha}$	=	momentum wheel torque
β	=	viscosity of the liquid fuel
θ, ψ	=	coordinates of the spherical pendulum
λ	=	proportionality factor in gyrostat equilibrium equation
μ	=	ratio of m_2 to the total mass, $m_1 + m_2$
ν	=	nutation angle
τ	=	time duration, s
Ω	=	angular velocity of the momentum wheel
$\boldsymbol{\omega}$, $\boldsymbol{\omega}_e$	=	angular velocities of the spacecraft and the Earth

Subscripts and Superscripts

e	=	Earth
f	=	final
o	=	pendulum pivot point
p	=	pendulum
r	=	pendulum length
s	=	steady state
to	=	turnover
w	=	wheel
$\times$	=	skew symmetric matrix used to form components of cross product of two vectors
zc	=	zero crossing
0	=	initial
1, 2, 3	=	x, y, and z components of the vectors

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Introduction

ANGULAR momentum exchange is commonly used to reorient the attitude of spacecraft. This reorientation maneuver is of particular interest due to the implementation ease and because it requires minimal propellant and sensors. Early studies of angular momentum exchange were investigated by Barba and Aubrun [1] and they considered the effect of a single internal wheel torque on the residual nutation angle and acquisition time of the spacecraft. Vadali and Junkins [2] improved the residual error at the end of the maneuver by introducing integrated control energy as a performance index. Hall [3,4] studied spin-up dynamics of dual spin spacecraft based on conservation of momentum and applied averaging theory to simplify multi-order nonautonomous equations. Recently, Kang and Lee [5] applied the angular momentum exchange technique to a satellite equipped with a spherical propellant tank and a momentum wheel fixed in the body frame. In practice, the wheel spin-up is limited by the available satellite power and the allowed time period of the maneuver is restricted by the timing of next step of the mission. The available power and desired acquisition time in early orbit are factors that size the momentum wheel torque motor. This wheel sizing directly influences the satellite mass budget, which impacts the launch cost. Therefore, the spin-up strategy and the associated acquisition time are of primary importance in satellite and mission design. However, since most previous studies assumed zero product of inertia of the spacecraft, the use of results has drawbacks when the products of inertia or slosh masses appear. In this paper, an analytic model to determine minimum wheel torques achieving turnover and desired maneuver times are presented. This new model, which includes propellant slosh and product of inertia terms, determines the wheel speed and acceleration for a given spacecraft configuration when the initial and steady-state body rates and the steady-state time are predetermined. It also determines the turnover wheel speed and corresponding time by incorporating the angular momentum of the initial and final spin axes. The analytical model is compared with numerical simulations to demonstrate its accuracy and appropriateness.

Equations of Motion

In this paper, the mathematical model is based on the idea developed in a previous study [5]. The physical model consists of a spacecraft main body, a momentum wheel, and a liquid fuel tank. The model for the spinning spacecraft uses an equivalent pendulum for the slosh mass [6,7]. If we define the moment of inertia and angular velocity of the spacecraft as $\mathbf{J}$ and $\boldsymbol{\omega}$, respectively, the angular momentum of the wheel as $\mathbf{h}_w$, and the angular momentum of the propellant as $\mathbf{h}_p$, then the total angular momentum of the system $\mathbf{H}$ will be

$$\mathbf{H} = \mathbf{J}\boldsymbol{\omega} + \mathbf{h}_w + \mathbf{h}_p \tag{1}$$

where $\mathbf{J} = \mathbf{I}_b + \mathbf{I}_w - \mu m_1 \mathbf{r}_p^\times \mathbf{r}_p^\times$, $\mathbf{h}_w = \mathbf{I}_w \boldsymbol{\Omega}$, $\mathbf{h}_p = \mu m_1 \mathbf{r}_p^\times \dot{\mathbf{r}}_p$. Using spherical coordinates as shown in Fig. 1, the location of the pendulum with respect to the tank center is expressed as $\mathbf{r} = r(-\sin\theta\sin\psi \quad \sin\theta\cos\psi \quad -\cos\theta)^T$ and the location of the pendulum with respect to C_1 is $\mathbf{r}_p = \mathbf{r}_o + \mathbf{r}$ where $\mathbf{r}_o = (0 \quad 0 \quad r_o)^T$. The time derivative of the vector $\mathbf{H}$ and Eq. (1) may be used to obtain the matrix equations:

$$\dot{\boldsymbol{\omega}} = \mathbf{J}^{-1}[\{\mathbf{H}^\times + \mu m_1 r^2(\dot{\mathbf{u}}_r^\times \mathbf{u}_p^\times + \mathbf{u}_p^\times \dot{\mathbf{u}}_r^\times)\}\boldsymbol{\omega} - \boldsymbol{\alpha} - \mu m_1 r^2 \mathbf{u}_p^\times \ddot{\mathbf{u}}_r] \tag{2}$$

$$\dot{\mathbf{h}}_w = \boldsymbol{\alpha} \tag{3}$$

where $\mathbf{u}_r$ is an unit vector of $\mathbf{r}$ and coincides with $\mathbf{u}_p$ when the tank location is at C_1.

The moment equation about the pendulum pivot point O is

$$\begin{aligned}\mu m_1 r^2 \mathbf{u}_r^\times[(\mathbf{E} + \mu m_1 r^2 \mathbf{u}_p^\times \mathbf{J}^{-1}\mathbf{u}_p^\times)\ddot{\mathbf{u}}_r - \mathbf{u}_p^\times \mathbf{J}^{-1}\{(\mathbf{H}^\times - \dot{\mathbf{J}})\boldsymbol{\omega} - \boldsymbol{\alpha}\} \\ + 2\boldsymbol{\omega}^\times \dot{\mathbf{u}}_r - \boldsymbol{\omega}^\times \mathbf{u}_p^\times \boldsymbol{\omega}] = \mathbf{T}_o\end{aligned} \tag{4}$$

where $\mathbf{T}_o$ is a torque about the pivot point O and can be replaced by a simple viscous torque model such as

$$\mathbf{T}_o = (-\beta r^2 \dot{\theta}\sin\psi \quad \beta r^2 \dot{\theta}\cos\psi \quad -\beta r^2 \dot{\psi}\sin^2\theta)^T \tag{5}$$

Analytical Model for Spin-Up

The best strategy for the wheel spin-up is to achieve synchronization between the spacecraft and the Earth, in other words to acquire an Earth-pointing attitude through the momentum exchange maneuver. However, the wheel spin-up is usually limited by the available satellite power. Especially during the launch and early operation phase before the solar array deployment, the power usage is extremely limited. Therefore, the achievable maximum momentum of the wheel is always constrained, and the spacecraft body spin rate even after the maneuver usually remains too large.

Angular Momentum Fixed System

In this paper, we assume that the moment of inertia about the x axis is maximum and the moment of inertia about the y axis minimum. Then let $\boldsymbol{\omega}_0$ and $\boldsymbol{\omega}_s$ be initial and steady-state body velocities, respectively, and $\boldsymbol{\Omega}_0$ and $\boldsymbol{\Omega}_s$ the corresponding wheel rates. Since the spacecraft state after the reorientation maneuver can be considered as a steady state, the corresponding conditions are $\dot{\boldsymbol{\omega}}_s = \dot{\boldsymbol{\Omega}}_s = \mathbf{0}$ and $\dot{\theta}_s = \ddot{\psi}_s = 0$. Assuming the effect of viscosity is ignorable, Eqs. (2) and (4) give

$$[\mathbf{H}_s^\times + \mu m_1 r^2(\dot{\mathbf{u}}_{rs}^\times \mathbf{u}_{ps}^\times + \mathbf{u}_{ps}^\times \dot{\mathbf{u}}_{rs}^\times)]\boldsymbol{\omega}_s - \mu m_1 r^2 \mathbf{u}_{ps}^\times \ddot{\mathbf{u}}_{rs} = \mathbf{0} \tag{6}$$

and

$$\ddot{\mathbf{u}}_{rs} + \boldsymbol{\omega}_s^\times(2\dot{\mathbf{u}}_{rs} - \mathbf{u}_{ps}^\times \boldsymbol{\omega}_s) = \mathbf{0} \tag{7}$$

Substitution of Eq. (7) into Eq. (6) yields

$$[\mathbf{H}_s^\times + \mu m_1 r^2(\dot{\mathbf{u}}_{rs}^\times \mathbf{u}_{ps}^\times - \mathbf{u}_{ps}^\times \boldsymbol{\omega}_s^\times \mathbf{u}_{rs}^\times - \mathbf{u}_{ps}^\times \dot{\mathbf{u}}_{rs}^\times)]\boldsymbol{\omega}_s = \mathbf{0} \tag{8}$$

If μ is small enough to be ignored, Eq. (8) can be reduced to a simple rotor-wheel problem:

$$\hat{\mathbf{H}}_s^\times \boldsymbol{\omega}_s = \mathbf{0} \tag{9}$$

where

$$\hat{\mathbf{H}}_s = (\mathbf{I}_b + \mathbf{I}_w)\boldsymbol{\omega}_s + \mathbf{I}_w \boldsymbol{\Omega}_s$$

Apparently, Eq. (9) is satisfied with any of the following conditions:

$$\boldsymbol{\omega}_s = \mathbf{0} \tag{10a}$$

$$\hat{\mathbf{H}}_s = \mathbf{0} \tag{10b}$$

$$\hat{\mathbf{H}}_s = \lambda \boldsymbol{\omega}_s \tag{10c}$$

where λ is called an equilibrium constant [8] or proportionality factor [9] in the gyrostat equilibrium equation. The conditions (10a) and (10b) are trivial because they require stationary motion or zero angular momentum. The third condition fits in our case and implies that the total angular momentum $\hat{\mathbf{H}}_s$ is parallel to $\boldsymbol{\omega}_s$. The equilibrium constant can be easily determined when the angular momentum and wheel speed are fixed. In words, by using the condition $\|(\mathbf{I}_b + \mathbf{I}_w)\boldsymbol{\omega}_s + \mathbf{h}_w\| = \|\hat{\mathbf{H}}_s\|$, the equilibrium constant can be found.

The conservation of angular momentum principle still holds for our original model such that

$$\|\mathbf{H}_0\| = \|\mathbf{H}_s\| \tag{11}$$

Based on Eq. (11) we can readily determine the wheel acceleration in terms of initial and final body rates for a given spacecraft configuration. Also, the initial and final body rates can be expressed in term of the Earth's rotation rate $\boldsymbol{\omega}_e$ such that

$$\boldsymbol{\omega}_0 = n_0 \boldsymbol{\omega}_e \tag{12}$$

$$\boldsymbol{\omega}_s = n_s \boldsymbol{\omega}_e \tag{13}$$

where n_0 and n_s are positive real numbers. Scaling the initial and steady-state angular velocity to the Earth spin rate is more convenient than using the actual angular velocities that may vary depending on mission phases. For example, n_s is one when a geosynchronous satellite is on station.

Example Spin-Up Scenario

Since the initial conditions for the pendulum and body motion are chosen to be $\theta_0 = \psi_0 = 0$, $\dot{\theta}_0 = 0$, $\ddot{\theta}_0 = \ddot{\psi}_0 = 0$, $\boldsymbol{\omega}_0 = (n_0\omega_e \quad 0 \quad 0)^T$, and $\boldsymbol{\Omega}_0 = \mathbf{0}$, the initial angular momentum of the system becomes

$$\mathbf{H}_0 = \mathbf{J}_0 \boldsymbol{\omega}_0 \tag{14}$$

After completion of the spin-up maneuver, the pendulum will move to its equilibrium point, $\theta_s = \pi/2$ and $\dot{\psi}_s = \text{const}$. The wheel

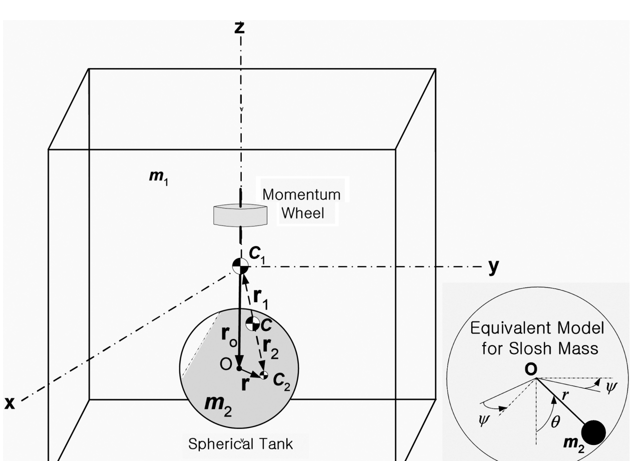

Fig. 1 Three-body system model.

also keeps in its steady state $\boldsymbol{\Omega}_s$. Then the steady-state angular momentum becomes

$$\mathbf{H}_s = \mathbf{J}_s \boldsymbol{\omega}_s + \mathbf{h}_{ws} + \mathbf{h}_{ps} \tag{15}$$

Using Eqs. (12–15), we can evaluate the wheel spin-up acceleration and the resulting wheel steady-state spin rate, turnover time, and spin-up time. The conservation of angular momentum principle gives a quadratic equation for the steady-state wheel spin rate. Eliminating the negative solution from the quadratic formula yields a steady-state solution with the first-order approximation in μ:

$$\Omega_{3s} = n_0 \omega_e \left(\frac{I_{11}^*}{I_{w33}}\right)\left[\left(\left(1-\left(\frac{n_s}{n_0}\right)^2\left(\frac{I_{23}}{I_{11}^*}\right)^2 + 2\hat{\mu} + \hat{\mu}^2\sigma_s^2\right)^{1/2} - \frac{n_s}{n_0}\left(\frac{I_{33}^*}{I_{11}^*} + \hat{\mu}\sigma_s\right)\right] + O(\hat{\mu}^2) \tag{16}$$

where $I_{ii}^* = I_{ii} + I_{wii}$, $\hat{\mu} = \mu m_1 r^2 / I_{11}^*$, and σ_s is defined as the ratio of the absolute pendulum angular velocity to the spacecraft angular velocity at steady state, i.e., $\sigma_s = (1 + \dot{\psi}_s / n_s \omega_e)$. If the product of inertia is small compared with the moment of inertia about the initial spin axis and $\hat{\mu}$ is also small enough to be ignored, Eq. (16) reduces further to

$$\Omega_{3s} \approx n_0 \omega_e \left(\frac{I_{11}^*}{I_{w33}}\right)\left[1 - \frac{n_s}{n_0}\frac{I_{33}^*}{I_{11}^*} - \frac{1}{2}\left(\frac{n_s}{n_0}\right)^2\left(\frac{I_{23}}{I_{11}^*}\right)^2\right] \tag{17}$$

which is a good approximation.

Spin-Up Time Duration

Once $\boldsymbol{\Omega}_0$ and $\boldsymbol{\Omega}_s$ for a given spacecraft configuration are determined, we are ready to select a wheel acceleration. In the above example, the wheel spin-up acceleration can be determined such that

$$\dot{\Omega}_3 = (\Omega_{3s} - \Omega_{30})/\tau_s \tag{18}$$

where $\tau_s = (t_s - t_0)$ is a spin-up time duration. Now we introduce a turnover time t_{to} at which ω_3 peak time coincides with the zero crossing time of ω_1. The turnover time is similar to the critical time introduced by Barba and Aubrun [2], at which the angle between the momentum vector and the final spin axis is close to zero. However, the zero crossing time of ω_1 does not correspond to the ω_3 peak time when the tank location moves away from the spacecraft center of mass. The spin-up time duration τ_s should be at least equal to or greater than the turnover time duration, $\tau_{\text{to}} = (t_{\text{to}} - t_0)$. Then, the wheel torque becomes

$$\boldsymbol{\alpha} = [0 \quad 0 \quad I_{w33}(\Omega_{3s} - \Omega_{30})/\tau_s]^T \tag{19}$$

Once the wheel acceleration is determined, τ_{to} can be determined by modifying Eq. (16) such that

$$\Omega_{3\text{to}} = n_0 \omega_e \left(\frac{I_{11}^*}{I_{w33}}\right)\left[\left(\left(1-\left(\frac{n_{\text{to}}}{n_0}\right)^2\left(\frac{I_{23}}{I_{11}^*}\right)^2 + 2\hat{\mu} + \hat{\mu}^2\sigma_{\text{to}}^2\right)^{1/2} - \frac{n_{\text{to}}}{n_0}\left(\frac{I_{33}^*}{I_{11}^*} + \hat{\mu}\sigma_{\text{to}}\right)\right] + O(\hat{\mu}^2) \tag{20}$$

and using

$$\tau_{\text{to}} = \frac{(\Omega_{3\text{to}} - \Omega_{30})}{(\Omega_{3s} - \Omega_{30})}\tau_s \tag{21}$$

Based on the conservation of momentum and the conservation of energy, the ratio n_{to}/n_0 in Eq. (20) can be approximated as unity. In addition, the turnover time becomes the steady-state time when the final spacecraft rate is chosen such that $n_{\text{to}} = n_s$. As seen in Eqs. (16) and (20), the slosh mass and the product of inertia are appeared as parameters which determine the wheel speeds and therefore the turnover time.

Numerical Example

In this example, we assume that the center of the momentum wheel is located on C_1, and $r_o/r = 0$. The following basic data are used throughout all simulations in this paper:

$$m_1 = 650 \text{ kg}, \qquad m_2 = 0.076 \cdot m_1, \qquad I_{11} = 500 \text{ kg} \cdot \text{m}^2$$
$$I_{22} = 0.8 I_{11}, \qquad I_{33} = 0.88 I_{11}, \qquad I_{23} = -0.014 I_{11}$$
$$I_{12} = I_{13} = 0, \qquad I_{w11} = I_{w22} = 0.0002 I_{11}$$
$$I_{w33} = 0.00034 I_{11}, \qquad I_{w12} = I_{w23} = I_{w13} = 0$$
$$r = 15 \text{ cm}, \qquad n_0 = 7200, \qquad n_s = 2n_0/3$$

The wheel spin-up profile is as follows:

$$\Omega_1 = \Omega_2 = 0 \qquad 0 \le t < t_f$$

$$\text{and} \quad \Omega_3 = \begin{pmatrix} 0 & t \le t_0 \\ (\Omega_{3s} - \Omega_{30})(t - t_0)/\tau_s & t_0 < t \le t_s \\ \Omega_{3s} & t_s < t \le t_f \end{pmatrix}$$

Steady-state speed of the pendulum about the spacecraft spin axis depends on its initial conditions and spin-up strategy during the maneuver. However, here we assumed that it has a 10% slower retrograde motion with respect to the spacecraft. We also assume that the spacecraft is initially spinning about the axis of maximum moment of inertia and the wheel axis is aligned with the axis of intermediate moment of inertia. The wheel is then accelerated to rotate relative to the spacecraft body. The spin-up of the wheel transfers momentum from the body to the wheel, thus arriving at a configuration in which the spacecraft's intermediate moment of inertia axis is reoriented and aligned with the angular momentum vector. The speed of the wheel remains zero until $t = t_0$, then increases linearly until it reaches Ω_{3s} at $t = t_s$ s , and finally remains constant. For a given wheel spin-up scenario, we determined $\Omega_{3s} = 637.70$ rad/s by using Eq. (16). The wheel spin-up time duration can be chosen based on the output performance of the momentum wheel and the allowable time period of maneuver. In this paper, seven different spin-up time durations are chosen, and corresponding wheel accelerations and turnover time durations are evaluated. As given in Table 1, taking a shorter spin-up time (steady-state time or maneuver time) requires a larger acceleration and a shorter turnover time duration but induces larger nutation angles while longer spin-up times yield smaller nutation angles.

Figure 2 displays time histories of the wheel acceleration and corresponding body rate of rotation for Case 1. It is shown that the zero crossing value of ω_1 and the maximum value of ω_3 occur at the turnover time. Figure 3 is a graphical representation of the wheel

Table 1 Wheel acceleration and turnover time over wheel spin-up duration

Parameter	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7
τ_s, s	1800	2400	3000	3600	4200	4800	5400
$\dot{\Omega}_3$, rad/s^2	0.3542	0.2657	0.2125	0.1771	0.1518	0.1328	0.1180
$\tau_{to,\text{ anal}}$, s	521.1	694.9	868.6	1042.3	1216.1	1389.8	1563.6
$\tau_{to,\text{ num}}$, s	550.0	723.6	896.4	1069.2	1242.0	1414.8	1587.6
ν_s, rad[a]	0.1201	0.1067	0.1033	0.0901	0.0836	0.0812	0.0761

[a]mean value of nutation angles evaluated at each integration time step after turnover time.

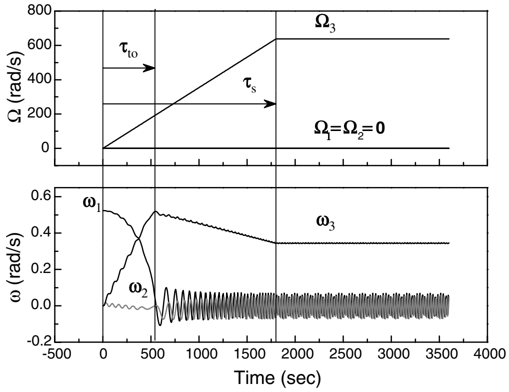

Fig. 2 Wheel spin-up and body angular rates.

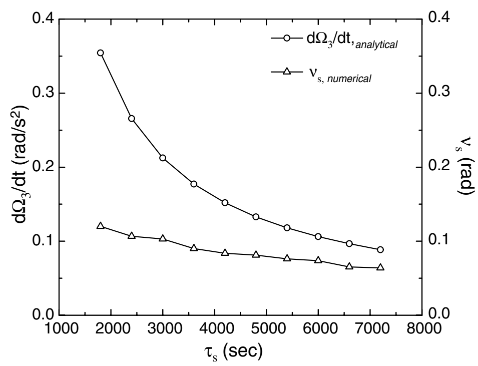

Fig. 3 Wheel acceleration and nutation vs steady-state time durations.

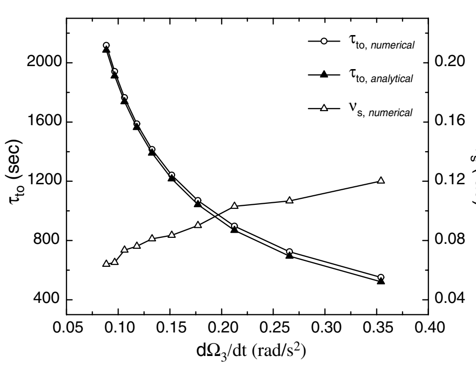

Fig. 4 Effect of wheel acceleration on turnover time and nutation.

spin-up time durations, accelerations, and mean nutation angles as given in Table 1. As explained before, choosing a longer spin-up time duration is better for smaller acceleration and nutational motion unless the mission schedule is tight. The effect of wheel acceleration on the turnover time duration and the averaged steady-state nutation angle is shown in Fig. 4. The predicted values of the turnover time duration by the analytical method match the numerical values within a few percent, and discrepancy between predicted and numerical values reduces as smaller wheel accelerations are chosen. The graph also shows that a larger wheel acceleration causes a greater steady-state nutation angle.

Conclusions

An analytical method to develop an appropriate wheel spin-up strategy using the momentum transfer technique has been studied. The resulting method considers the effect of slosh mass contained in the spacecraft and requires knowledge of initial and desired steady-state body rates. Once the initial and steady-state body rates are selected, the wheel steady-state spin rate can be analytically determined. The wheel spin-up acceleration can also be obtained by simply dividing the steady-state spin rate of the wheel by the spin-up time duration, which is selected by considering the amount of available satellite power and the allowed time before the start of the next step of the mission. The spin-up time duration, which determines the wheel spin-up acceleration, should be greater than the turnover time duration of the spacecraft body. The analytical solutions derived in this paper were verified through comparisons to numerical solutions and agreed within a few percent. This analytical model can be used as an easy-to-use procedure for reorienting similar spacecraft.

References

[1] Barba, P. M., and Aubrun, J. N., "Satellite Attitude Acquisition by Momentum Transfer," *AIAA Journal*, Vol. 14, No. 10, Oct. 1976, pp. 1382–1386.
doi:10.2514/3.7228

[2] Vadali, S. R., and Junkins, J. L., "Spacecraft Large Angle Rotational Maneuvers with Optimal Momentum Transfer," *Journal of the Astronautical Sciences*, Vol. 31, No. 2, 1983, pp. 217–235.

[3] Hall, C. D., and Rand, R. H., "Spinup Dynamics of Axial Dual-Spin Spacecraft," *Journal of Guidance, Control, and Dynamics*, Vol. 17, No. 1, 1994, pp. 30–37.
doi:10.2514/3.21155

[4] Hall, C. D., "Spinup Dynamics of Gyrostats," *Journal of Guidance, Control, and Dynamics*, Vol. 18, No. 5, 1995, pp. 1177–1183.
doi:10.2514/3.21522

[5] Kang, J. Y., and Lee, S., "Attitude Acquisition of a Satellite with a Partially Filled Liquid Tank," *Journal of Guidance, Control, and Dynamics*, Vol. 31, No. 3, May 2008, pp. 790–793.
doi:10.2514/1.31865

[6] Kang, J. Y., and Cochran, J. E., Jr., "Resonant Motion of a Spin-Stabilized Thrusting Spacecraft," *Journal of Guidance, Control, and Dynamics*, Vol. 27, No. 3, 2004, pp. 356–364.
doi:10.2514/1.10331

[7] Kang, J. Y., and Cochran, J. E., Jr., "Stability Criteria of Slosh Motion with Periodicity in a Spinning Spacecraft," *Journal of Guidance, Control, and Dynamics*, Vol. 28, No. 3, 2005, pp. 562–567.
doi:10.2514/1.13580

[8] Oh, H. S., "Spinup Maneuver and Nonlinear Active Nutation Damping," *Journal of Guidance, Control, and Dynamics*, Vol. 19, No. 3, 1996, pp. 704–706.
doi:10.2514/3.21678

[9] Hughes, P. C., *Spacecraft Attitude Dynamics*, Wiley, New York, 1986, pp. 165–171.